Intergenerational risk-sharing and risk-taking of a pension fund

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Abstract

Pension funds can provide some intergenerational risk-sharing by smoothing shocks on returns of assets, using their financial reserves efficiently. In addition to the direct benefit of improved time diversification, this better allocation of risk allows the fund to take better advantage of the equity premium, thereby yielding an additional benefit for consumers. In this paper, our aim is twofold. First, we characterize the socially efficient policy rules of the plan in terms of portfolio management, capital payment to retirees, and dividends payment to shareholders. We examine both the first-best rules and the second-best rules in which the fund is constrained by a solvency ratio and by a guaranteed minimum return. Second, we measure the social surplus of the system compared to a situation where each generation would save and invest in isolation for its retirement.

Keywords: dynamic portfolio choice, pension plan, retirement, intergenerational risk-sharing, financial intermediation.
1 Introduction

As is well-known, when contingent markets are complete, the competitive equilibrium yields a Pareto-efficient allocation of risk in the economy. However, markets are notably incomplete in reality, in particular because current generations cannot trade risk with those who are not borne yet. For example, as observed first by Gordon and Varian (1988), current workers who accumulate wealth for their retirement are unable to share their financial risk with future workers. Looking retrospectively, this risk exposure can be large for those who heavily invested in equity. In the recent past, many of them had to drastically reduce their standard of living and to work longer after a sizeable downturn of financial markets, as they occur periodically. This inability of markets to allocate risk efficiently has been used to advocate public intervention, in particular in the form of pay-as-you-go (PAYG) systems. However, the European experience about PAYG systems tells us that organizing such risk-sharing efficiently is subject to a strong political constraint.

Financial intermediaries, and more particularly life insurers, have a long tradition to reallocate risk across different generations of contracts through a discretionary strategy of distribution of benefits to their customers over time. Typically, they withhold some of the benefits generated by their portfolio of assets when financial markets are bullish, and these additional reserves are used to improve the distribution of benefits to policyholders in bad time. In the past, this system was made possible in particular by allowing insurers to value their assets at their purchased price rather than at their so-called "fair" value. It is widely believed that recent reforms in Europe concerning the accounting standards of financial intermediaries puts the system at risk of not being able to perform this intergenerational sharing of financial risk in the future.

In this paper, we describe an explicit intergenerational risk-sharing mechanism based on a transparent funded pension system, and we measure the social surplus that this system generates compared to a system of personal savings accounts. We consider an overlapping generation model in which each worker contributes to the fund during $n$ years before retiring. The level of defined contribution is exogenous. Each year, a new generation of workers

\[ \text{Shiller (2003) discusses extensively the potential gains of improving intergenerational risk-sharing.} \]
starts saving in the fund, whereas another generation leaves the fund with a capital payment that is endogenously determined. The reserves of the insurer can be invested into two assets, one of which is risk free. By providing capital to the fund, the insurer is able to offer portfolio insurance to consumers that take the form of a minimum return on their savings. The problem is to determine in each year the optimal portfolio of assets, the payment of benefits to the new pensioners, and the distribution of dividends to the shareholders of the insurance company. The insurer is constrained by a solvency ratio, which guarantees that the current market value of assets is always larger than the current market value of liabilities to the contributing workers.

The main contribution of the paper is to describe the socially efficient portfolio management and benefit distribution of the fund. Because the problem cannot be solved analytically due to both the solvency constraint and the guarantee of a minimal return on savings, we rely on a numerical optimization procedure. The socially efficient strategy is described by policy functions, which link respectively the portfolio allocation, the distribution of benefits to new pensioners and the distribution of dividends to shareholders to the level of financial reserves of the fund. Many features of these efficient policy functions are noteworthy. First, the share of the fund’s wealth invested in the risky asset is an inverted U-shaped function of the fund’s wealth. This means in particular that the fund drastically reduces its portfolio risk after a loss, when the solvency constraint becomes an issue. Second, the benefit distributed to new pensioners is increasing and concave in the market value of the fund’s assets. When the solvency problem is an issue, the distributed benefit above the contractual minimum is very sensitive to the financial situation of the fund, which means that the intergenerational risk-sharing does not work in these circumstances. On the contrary, when the fund is wealthy, the benefit distributed to the new pensioners is much less sensitive to shocks on the market value of the fund’s assets. In other words, the intergenerational sharing of risk is possible only when the fund has enough wealth to smooth financial shocks across different generations of workers. Notice that the fact that shareholders of the insurance company bear the downside risk of financial markets, dividends are very volatile and low when reserves are in the red zone. Shareholders are compensated for this provision of portfolio insurance to policyholders by a competitive premium on their initial investment.

We also measure the welfare gain of the improved risk-sharing in the econ-
omy. In order to perform this welfare analysis, we compare three long-term saving schemes. The first two schemes are compared by Ball and Mankiw (2001) in the case of an exogenous assets portfolio. The first saving scheme, which is presented in section 2, is based on personal retirement accounts. Workers determine their optimal portfolio risk in each period of their life, knowing that there will be no solidarity across generations in case of a financial downturn. We show that it is optimal in that case for workers to invest their entire pension saving in the risky asset in the first half of their career. Then, they should gradually rebalance their portfolio in favor of the risk-free asset, to finish with a portfolio containing 60% invested in that asset. We show that following this optimal dynamic portfolio strategy is equivalent in terms of expected utility of pension wealth to a sure investment of lifetime savings at an interest rate of 3.25%. The second long-term savings scheme examined in section 3 yields a first-best intergenerational risk-sharing. It maximizes a discounted sum of the expected utility of future generations subject to an intertemporal budget constraint. The first-best strategy consists in investing a constant share of the fund’s total wealth in the risky asset, and to distribute a constant share of the fund’s total wealth to retirees. The fund’s total wealth is the sum of the current financial reserves and the net present value of future workers’ contributions. The first-best solution yields an intergenerational welfare that is equivalent to the one that would be obtained by investing pension savings at a risk-free interest rate of 4.85%. This shows the potentially large welfare gain of intergenerational risk-sharing. However, the main problem of the first-best solution is the possible fast exhaustion of the financial reserves of the scheme and the unwillingness of future generations to participate to the scheme in these circumstances. This is why we examine in section 4 the more realistic second-best pension scheme in which the pension fund is constrained by a solvency ratio and by the requirement of a minimum rate of return of 0% to its members. The second-best scheme yields an intergenerational welfare that is equivalent to a 3.76% sure rate of returns on savings. The constraints imposed in the second-best scheme are not enough to guarantee that future generations will always prefer to participate than to save on their own for their retirement. However, we show that a reasonable tax incentive to participate is sufficient to organize the sustainability of the system in the worst case scenario.
2 Optimal individual saving accounts

The economy is composed of overlapping generations of a fixed size that is normalized to unity. Each generation contributes to a pension fund during \( n = 30 \) years. The yearly contribution \( y \) is paid out by the active workers at the beginning of the year. We are in a context of defined contributions, which implies that \( y \) is exogenous to the model. At the end of the \( n \)th year, the old generation receives a benefit whose level is endogenous to the model. This retiring generation is replaced by a young generation which starts contributing to the fund. This means that \( n \) contributing generations coexist in the fund, which receives \( ny \) monetary units at the end of each period. The welfare of a generation is measured by the expected utility that it obtains from consuming the pension benefit at the retirement age. The workers’ utility function \( u \) is increasing and concave in the pension benefit \( b \). In this paper, we assume that workers have constant relative risk aversion equaling \( \gamma \), so that \( u(b) = b^{1-\gamma}/(1-\gamma) \). We take a relatively conservative position by assuming that \( \gamma = 5 \).

There are two financial assets in the economy, one of which is risk-free. Asset returns are unpredictable. The gross yearly return of the risk free asset is constant over time and is equal \( R \). In our calibration, we consider an interest rate of 2\% \( (R = 1.02) \). The excess return of the risky asset in year \( t \) is denoted \( \bar{x}_t \). There is no serial correlation of returns over time, so that \( \bar{x}_1, \bar{x}_2, \ldots \) are independent and identically distributed. \( R \) and \( \bar{x} \) are exogenous. In particular, we assume that they are not affected by the architecture of the pension system, as would be the case in a small open economy. We calibrate our model with the empirical distribution of the real yearly return of the SP500 in excess of the risk-free rate over the period 1963-1994. It is depicted in Figure 2. The expected real excess return is \( E\bar{x} = 3.9\% \), and the standard deviation is \( \sigma_x = 13.6\% \).

As a benchmark, we examine an economy in which there is no risk-sharing. The fund can be interpreted in that case as a collection of \( n \) individual saving accounts. In this model, each generation can be considered in autarcy. At the beginning of each of the \( n \) contribution years, the generation has to determine how much of their reserves to invest in the risky asset, the

\[ \text{This implies that workers are ready to pay as much as 2.4\% of their wealth to eliminate a fifty-fifty risk to gain or loose 10\% of their wealth.} \]
remaining being invested in the risk-free asset. Let \( \alpha_t \) denote the monetary investment in the risky asset at the beginning of the \( t \)th contribution year, \( t = 1, \ldots, n \). The financial reserve of a generation from contribution year \( t - 1 \) to contribution year \( t \) is denoted \( w_t \). The individual dynamic portfolio problem can be written as

\[
U^{\text{aut}} = \max_{\alpha_1, \ldots, \alpha_n} E u(\tilde{b})
\]

s.t. \[
\begin{align*}
  w_1 &= 0; \\
  w_{t+1} &= R(w_t + y) + \alpha_t \tilde{x}_t, \quad t = 1, \ldots, n; \\
  \tilde{b} &= w_{n+1}; \\
  \alpha_t &\leq w_t + y, \quad t = 1, \ldots, n.
\end{align*}
\]

Notice that we impose a no-borrowing constraint which means that agents cannot invest more than their current financial reserves in the risky asset.

Because we assume constant relative risk aversion, the solution to this dynamic portfolio problem is well-known since Merton (1969) and Samuelson (1969). Define the pension wealth of the generation at the beginning of its contribution in year \( t \) as

\[
z_t = w_t + \sum_{i=0}^{n-t} R^{-i} y.
\]

The pension wealth equals the sum of the current financial reserves and of the net present value of the flow of future contributions discounted at the risk-free rate. Define also the constant \( a^* \) which is the unique root of the following equation:

\[
E \tilde{x}(1 + a^* \tilde{x})^{-\gamma} = 0.
\]

Under the assumption that \( E \tilde{x} \) is positive, this root \( a^* \) is positive. The optimal investment strategy is then determined by a policy function \( \alpha^{\text{aut}} \) such that the investment in the risky asset at the beginning of the contribution year \( t \) equals

\[
\alpha_t = \alpha^{\text{aut}}(z_t, w_t) = \min(a^* z_t, w_t).
\]

As long as the no-borrowing constraint is not binding, it is optimal to invest a constant share \( a^* \) of the pension wealth \( z_t \) in the risky asset. This illustrates
the well-known fact that myopia is optimal under constant relative risk aversion. However, it is standard to express the optimal investment rule as the share of current wealth $w_t$ rather than of total wealth $z_t$ invested in the risky asset. Analyzed in these terms, the optimal portfolio risk expressed by $\alpha_t/w_t$ is decreasing with age, because $w_t$ is typically increasing with age. This is the argument proposed by Bodie, Merton and Samuelson (1992) to justify the standard recommendation to reduce portfolio risk when growing older. When the no-borrowing constraint is binding, the generation invests 100% of its financial reserves in the risky asset. This constraint is likely to be binding early in the contribution period of consumers, because the pension wealth $z_t$ is much larger than the financial reserve $w_t$ at that time.

Under our assumption, the target share of pension wealth that should be invested in stocks is $a^* = 39.6\%$. In Figure 3, we selected randomly ten scenarios of stock returns over the 30 years, and we depicted the evolution of financial reserves of a consumer following the optimal investment strategy. In Figure 4, we represented the share of financial reserves invested in stocks over time in these 10 random scenarios. It clearly appears that the uncertainty on the final wealth available to the retiree is large, in spite of the important rebalancement of portfolios in favor of the risk-free asset as investors get closer to retirement. This shows that there is much room for intergenerational risk-sharing in this economy. It is noteworthy that, as explained by Samuelson (1963), there is no time-diversification of the portfolio risk going on in this personal retirement system. The portfolio risk taken in year 1 is not diversified away by the portfolio risk taken over the $n-1$ remaining years. Diversification could exist here only through an intergenerational scheme, in which one generation would exchange half of its lifetime portfolio risk with half of the portfolio risk borne by another (distant) generation.

Using backward induction, we computed numerically $U^{aut}$, the optimal expected utility of the final pension wealth as defined in (1). We define the certainty equivalent final wealth $B^{aut}$ as the sure pension wealth that yields the same expected utility for the retiree than the random pension wealth $\tilde{b}$ that is obtained by following the optimal portfolio strategy (4): $u(B^{aut}) = U^{aut}$. We obtain $B^{aut} = 51.13y$. The constant flow of saving optimally invested in financial markets over the $n = 30$ years preceding retirement yields the same expected utility for the retiree than receiving a capital of 51 times the yearly saving at retirement age. We can also define the certainty equivalent rate of return on saving as the interest rate $r^{aut}$ on
saving that yields the expected utility $U^{aut}$:

$$u \left( \sum_{t=1}^{30} \frac{y}{(1 + r^{aut})^t} \right) = U^{aut}. \quad (5)$$

We obtain $r^{aut} = 3.25\%$. The ability to invest in stocks raises the certainty equivalent rate of return of savings from the risk-free rate of $R - 1 = 2\%$ to 3.25%. This return is still far away from the expected rate of equity, which is assumed in our calibration to be equal to 5.9%. In the next section, we examine whether we can improve the welfare of workers by diversifying risk among different generations of workers.

### 3 First-best intergenerational risk-sharing

Suppose that at time $t = 0$, the $n$ current generations agree to transfer their individual saving accounts to a pension fund that would be allowed to reallocate risks across generations. Let $Y_0$ denote the value of this initial transfer. It depends upon the realized returns of the risky asset during the last $n$ years. We hereafter assume that $Y_0$ equals the average wealth accumulated by the different generations coexisting in year $t = 0$, so that

$$Y_0 = \sum_{t=1}^{n} Ew_t, \quad (6)$$

where $Ew_t$ is the expected wealth accumulated by the generation in contribution year $t$ in autarchy. Using the model of the previous section, we obtained numerically that $Y_0$ equals 784$y$. In this section, we assume that contributions to the fund by future generations are compulsory. The net present value of these future contributions equals

$$K = \sum_{t=0}^{\infty} R^{-t} ny = \frac{nyR}{R - 1}. \quad (7)$$

In our calibration, we obtain that $K$ equals 1530$y$.

Let $w_t$ be the market value of all assets owned by the fund at the end of year $t-1$. At the beginning of year $t$, the fund collects the savings $y$ of the $n$

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3Without the no-borrowing constraint, the certainty equivalent pension wealth would equal 52.25 $y$, and the certainty equivalent rate of return on savings would equal 3.37%.
\( \alpha_t: \) investment in the risky asset
\( x_t: \) return of the risky asset
\( b_t: \) pension
\( ny: \) contributions
\( w_t \rightarrow \alpha_t \rightarrow x_t \rightarrow w_{t+1} \)

Figure 1: Financial flows in year \( t \).

contributing generations. It also distributes the benefit \( b_t \) to the generation which just retired at the end of the previous period. As before, \( \alpha_t \) is the money investment in the risky asset by the fund in year \( t \). In Figure 1, we represented these flows in year \( t \).

We measure the social welfare of the overlapping generations of workers by the discounted sum of the flow of expected utility generated by consuming their pension wealth at retirement. The discount factor \( \beta \) is fixed at 0.94 in the benchmark calibration.\(^4\) Consider the decision problem of the fund at the beginning of period 0. We look for the dynamic contingent strategy of the fund, both in terms of distribution \( b \) and of portfolio management \( \alpha \), that maximizes the welfare of the overlapping generations of workers:

\[
U^{fb} = \max E \left[ \sum_{t=0}^{\infty} \beta^t u(\tilde{b}_t) \right]
\]

s.t. \( w_{t+1} = R(w_t - b_t + ny) + \alpha_t \tilde{x}_t, \quad \forall t \geq 0; \)
\[
w_t + K \geq 0, \quad \forall t \geq 1,
\]

with \( w_0 = Y_0 \). Observe that we allow the fund to value the flow of future contributions, so that the market value of all ”assets” of the fund equals \( w_t + \)

\(^4\)This relatively low value of \( \beta \) has been selected in order to obtain a reasonable optimal growth rate of the reserves of the fund.
which is constrained to be nonnegative. It happens that the solution of
this program is well-known. The optimal strategy of the fund is characterized
by two policy functions \( b_t = b(w_t) \) and \( \alpha_t = \alpha(w_t) \). These
two functions can be interpreted as a contract among the different generations
that stipulates how much risk will be accepted, and how will these risks be
allocated over time. We have that

\[
b^f_b(w) = m(w + K) \quad \text{with} \quad m = 1 - \left( \beta R^{1-\gamma} E[1 + a^*\bar{x}]^{-\gamma} \right)^{1/\gamma}.
\]  \hspace{1cm} (9)

We assume that \( \beta \) is smaller than \( R^{\gamma-1}/E[1 + a^*\bar{x}]^{-\gamma} \), so that \( m \) is positive.\(^5\)

Every year, the fund should distribute to the retiring generation a constant
share \( m = 3.39\% \) of the market value \( w_t + K \) of all assets of the fund. This
means for example that a generation bears only 3.39\% of the portfolio risk
taken by the fund the year before this generation retires, thereby illustrating
the intergenerational risk-sharing of the scheme. Observe also that the first
generation, the one that retires during the year of the creation of the fund,
get a pension wealth equaling \( m(Y_0 + K) = 78.44y \). This is more than what
they just transferred to the fund, which was 61.9y.

The optimal portfolio strategy of the fund is given by the following rule:

\[
\alpha^f_b(w) = R(1 - m)a^*(w + K),
\]

The fund should invest a constant share \( R(1 - m)a^* = 39.0\% \) of its aggregate
wealth in the risky asset. Because the aggregate wealth includes the net
present value \( K \) of the flow of future contributions, this implies that the
demand for risky assets is much increased by the improved intergenerational
risk-sharing arrangement. Under the system of individual pension accounts
presented in the previous section, we can estimate numerically the average
demand for the risky asset to \( E[\sum_{t=1}^{\infty} \alpha^m_{at}] \approx 455.8y \). At \( t = 0 \), the demand
for the risky asset jumps to \( \alpha^{fb}(Y_0 + K) = 902.5y \). Because risks are better
diversified in the economy, it is socially efficient to accept more risk in the
economy.

We can also examine the dynamics of the fund’s reserves. If the fund follows
the distribution and investment strategy presented above, the expected
total wealth of the fund has the following property:

\[
E[w_{t+1} + K] = R(w_t - b_t + ny) + \alpha_t E\bar{x} + K
\]

\(^5\)If \( \beta \) is larger than \( R^{\gamma-1}/E[1 + a^*\bar{x}]^{-\gamma} \), this problem has an unbounded solution.
\[ R(w_t + K) - Rm(w_t + K) + R(1 - m)a^*(E\bar{x})(w_t + K) \]
\[ = R(1 - m)[1 + a^*E\bar{x}](w_t + K). \]

It implies that the expected growth rate of the fund’s aggregate wealth is constant over time. It equals \( R(1 - m)[1 + a^*E\bar{x}] - 1 = 0.06\% \) per year. It is noteworthy that the expected growth rate of the fund’s reserves is positive, so that, seen from the veil of ignorance at date \( t = 0 \), the expected utility of the different generations is increasing over time.

Finally, we can measure the impact of the first-best intergenerational risk-sharing on the intergenerational welfare. We have that

\[ U^{fb} = \frac{m^{-\gamma}(Y_0 + K)^{1-\gamma}}{1 - \gamma}. \] 

It is easier to measure the certainty equivalent constant benefit \( B^{fb} \) distributed to all future generation that yields the same intergenerational welfare: \( U^{fb} = u(B^{fb})/(1 - \beta) \). In our calibration, we obtain \( B^{fb} = 67.89 \) y, a benefit that should be compared to what generations obtain in autarcy, \( B^{aut} = 51.13 \) y. In terms of certainty equivalent interest rate on savings, using a formula equivalent to (5), we obtain that \( r^{fb} = 4.85\% \). This is a considerable increase compared to the certainty equivalent interest rate obtained in autarcy: \( r^{aut} = 3.25\% \).

We conclude that the reform is Pareto-improving under the veil of ignorance: given the information available at \( t = 0 \), all current and future generations enjoy a larger expected utility than under autarcy. However, as is usual in most dynamic risk-sharing arrangements, there is a possible commitment problem ex-post. Because of the large portfolio risk taken by the fund, successive adverse shocks on financial markets can dramatically reduce the reserves of the fund in such a way that young workers may prefer to switch back to an individual pension system. To examine this question, we determine the certainty equivalent pension benefit \( b^{fb}(w) \) for a young worker who starts contributing to the system in year \( t \) given the financial reserve \( w \) of the fund at that date:

\[ u\left(b^{fb}(w)\right) = E\left[u(\bar{h}_{t+n}) \middle| w_t = w\right] = m^{1-\gamma}\left[R^{1-\gamma}(1 - m)^{1-\gamma}E(1 + a^*\bar{x})^{1-\gamma}\right]^n\frac{(w + K)^{1-\gamma}}{1 - \gamma}. \] 

A commitment problem arises for the young generation entering into the system if \( b^{fb}(w) \) is smaller than the certainty equivalent pension in autarcy.
\( B^{aut} = 51.13 \ y. \) We derived numerically that this happens when reserves \( w \) are smaller than 325.5\( y. \) In spite of the facts that the expected growth rate of reserves is positive and that the initial reserves be much larger than this minimum, such a situation is likely to occur in the future. We illustrate this point in Figure 5, where ten random scenarios of asset returns have been drawn over a century. Among these ten histories, the fund faces a commitment problem in two of them. In these two scenarios, the problem is severe and persistent. Financial reserves remain persistently in negative grounds, yielding a very low return on contributions for workers.

4 Second-best intergenerational risk-sharing

The first-best intergenerational risk sharing is hardly politically sustainable if a succession of negative shocks on financial markets arise early in the life of the fund. In particular, the fund may need to serve a negative return on contributions in order to restore its attractiveness in the future. In this section, we add a few additional constraints to the system in order to make it less sensitive to this political sustainability problem.

4.1 The second-best constraints

The key additional element that we add in the system is a constraint on the minimum return that must be served to workers. The fund guarantees a minimum gross return \( R_{\text{min}} \) to workers on their savings. This guaranteed minimum return of the fund can be interpreted as a portfolio insurance scheme. This implies that the minimum benefit paid to the new retirees is

\[
b_{\text{min}} = \sum_{i=1}^{n} yR_{\text{min}}^i.
\]  

(12)

Because of this guarantee, the fund is also constrained to maintain in all time and in all states a minimum capital \( w_{\text{min}} \) which is the guaranteed capital accumulated by the current contributing generations. The solvency check is made at the beginning of the year, after pensions and dividends have been paid, but before contributions have been received. This means that if, for any reason, the fund has to interrupt its activities, the current market value of its assets would be enough to repay the contributions paid in the past by
the currently active workers, in addition to a return $R_{\min}$ on these savings. This implies that $w_{\min}$ equals

$$w_{\min} = \sum_{j=0}^{n-1} \left[ \sum_{i=1}^{n-j} yR_{\min}^j \right] = \frac{yR_{\min}}{R_{\min} - 1} \left( \frac{R_{\min} (R_{\min} - 1)}{R_{\min} - 1} - n \right).$$

(13)

The last inequality holds only if $R_{\min}$ is not equal to unity. When $R_{\min} = 1$, we simply have that $w_{\min} = yn(n + 1)/2$. In our benchmark calibration, we assume that the fund must guarantee a minimum return of 0% on the workers’ contributions, i.e., $R_{\min} = 1$. Combining this with $n = 30$, we obtain that the minimum capital requirement is $w_{\min} = 465y$. The minimum benefit paid to retirees is $b_{\min} = 30y$.

We consider an economy in which a shareholders company manages the pension fund. Shareholders live forever and have an increasing and concave utility function $v$ on the dividends $d_t$ that are paid to them by the fund at the beginning of each year. To make things comparable with the first-best solution, we assume that shareholders have the same attitude towards risk than workers: $v \equiv u$. At date $t = 0$, the new pension scheme is created and is funded with the combination of equity $e_0$ from shareholders and of initial contributions $Y_0$ from the $n$ generations of workers living at that time, so that $w_0 = e_0 + Y_0$. As in the previous section, we assume that $Y_0$ equals $784y$. There is a competition among insurers to determine who will manage the fund in the future. This implies that $v_0$ equals the expected utility that insurers can obtain by investing $e_0$ directly on financial markets, or that

$$v_0 = \max E \left[ \sum_{t=0}^{\infty} \beta^t v(\tilde{d}_t) \right]$$

s.t.

$$e_{t+1} = R(e_t - d_t) + \alpha_t \tilde{x}_t, \quad \forall t \geq 0;$$

$$e_t \geq 0, \quad \forall t \geq 1.$$

From the previous section, we know that this problem has an analytical solution, yielding

$$v_0 = \frac{m^{-\gamma} e_0^{1-\gamma}}{1 - \gamma}. \quad (14)$$

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The capital $e_0$ invested by shareholders at date $t = 0$ is exogenous. We assume that $e_0$ is equal to 100y, an investment that should be compared to the initial workers' contribution $Y_0 = 784y$.

Consider the decision problem of the fund at the beginning of period 0, given its financial reserves $w_0 = e_0 + Y_0 \geq w_{\min}$. We look for the dynamic contingent strategy of the fund, both in terms of distribution and portfolio management, that maximizes the welfare of the overlapping generations of workers, subject to the participation constraint (16) of the shareholders of the fund, the minimum benefit constraint, the solvency constraint and the no-borrowing constraint:

$$U^{sb} = \max E \left[ \sum_{t=0}^{\infty} \beta^t u(b_t) \right]$$

s.t. $v_0 \leq E \left[ \sum_{t=0}^{\infty} \beta^t v(a_t) \right]$; (16)

$$w_{t+1} = R(w_t - b_t - d_t + ny) + \alpha_t \tilde{x}_t, \quad \forall t \geq 0;$$

$$b_t \geq b_{\min}, \quad \forall t \geq 0;$$

$$w_t - b_t - d_t \geq w_{\min}, \quad \forall t \geq 0;$$

$$\alpha_t \leq w_t - b_t - d_t + ny, \quad \forall t \geq 0.$$ (20)

This maximization program is well-behaved and has a feasible solution if $R$ is larger than $R_{\min}$. The optimal solution is referred to as the second-best strategy. Because this program has no analytical solution, we hereafter relies on a numerical algorithm to describe the second-best strategy. Using Mathematica©, we solve the problem by backward induction using 200 iterations, starting from the first-best value function. In Table 1, we summarize the value of the exogenous parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>5</td>
<td>constant relative risk aversion of workers and shareholders;</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.94</td>
<td>discount factor;</td>
</tr>
<tr>
<td>$\tilde{x}$</td>
<td>3.9%</td>
<td>excess return of the risky asset ($E\tilde{x} = 3.9%$);</td>
</tr>
<tr>
<td>$R$</td>
<td>1.02</td>
<td>gross interest rate;</td>
</tr>
<tr>
<td>$R_{\min}$</td>
<td>1.00</td>
<td>gross guaranteed minimum return on contributions;</td>
</tr>
<tr>
<td>$w_{\min}$</td>
<td>465y</td>
<td>minimum value of assets of the fund;</td>
</tr>
<tr>
<td>$Y_0$</td>
<td>784y</td>
<td>initial contribution of workers at $t = 0$;</td>
</tr>
<tr>
<td>$e_0$</td>
<td>100y</td>
<td>initial equity of the insurance company at $t = 0$.</td>
</tr>
</tbody>
</table>
4.2 The allocation of risk between workers and shareholders

It is immediate that we can rewrite program (15) as follows

\[ U^{sb} = \max \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \tilde{u}(\tilde{c}_t) \right] \]  

(21)

\[ \text{s.t. } w_{t+1} = R(w_t - c_t + ny) + \alpha_t \tilde{x}_t, \quad \forall t \geq 0; \]
\[ w_t - c_t \geq w_{\text{min}}, \quad \forall t \geq 0; \]
\[ \alpha_t \leq w_t - c_t + ny, \quad \forall t \geq 0, \]

where function \( \tilde{u} \) is defined as

\[ \tilde{u}(c) = \max_b u(b) + \lambda v(c - b) \quad \text{s.t. } b_t \geq b_{\text{min}}. \]

(22)

The Lagrange multiplier \( \lambda \) must be selected in such a way that the participation constraint (16) be satisfied as an equality. By decomposing program (15) into the two above programs, we disentangle the dynamic accumulation and risk-taking problem (21) of the fund from the purely static problem (22). Program (22) is a standard cake-sharing problem which determines how the total yearly payment \( c \) of the fund in any given year should be shared between a benefit \( b \) to workers and a dividend \( d = c - b \) to shareholders. Because \( u \) and \( v \) exhibit the same constant relative risk aversion, the first-best risk-sharing rule would be to allocate a fixed proportion \( k \in [0, 1] \) of the payment to workers: \( \tilde{b}_t = k \tilde{c}_t \), with \( k = (1 + \lambda^{1/\gamma})^{-1} \). As parameter \( \lambda \), parameter \( k \) is a function of \( u_0 \).

The presence of the guaranteed minimum return of workers’ contributions implies that the second-best risk-sharing rule between workers and shareholders takes the following form:

\[ \tilde{b}_t = \max(b_{\text{min}}, k \tilde{c}_t). \]

(23)

The second-best contract offers a constant share \( k \) of the total yearly payment of the fund to the retiring workers, as long as this pension benefit is larger
than the guaranteed minimum $b_{\text{min}}$. The dividend paid to shareholders equals $d_t = \min(\tilde{c}_t - b_{\text{min}}, (1-k)\tilde{c}_t)$. Shareholders are much exposed to the downside risk on the total yearly return, as is usual for sellers of portfolio insurance. It is noteworthy that the indirect utility function $\bar{u}$ exhibits a constant relative risk aversion $\gamma = 5$ as long as the minimum return constraint is not binding. When this constraint is binding, workers do not participate to the sharing of the portfolio risk, thereby raising the relative risk aversion of the syndicate. This relative risk aversion goes up to plus infinity when the total payment $c$ goes down to $b_{\text{min}}$.

In our benchmark calibration, the Lagrange multiplier $\lambda$ equals $0.4039 \times 10^{-6}$, so that the share $k$ of the total yearly payment accruing to retiring workers equals 95.0% of the total yearly payment. The remaining 5% is distributed to shareholders of the insurance company as a return for their initial risky investment.

### 4.3 The benefit policy and the dividend policy

In this section, we examine the second-best strategy of benefit payment to the retiring generation. This benefit is a function of the level of financial reserves of the fund at the retirement date: $b_t = b^{fb}(w_t)$. We have seen that the first-best benefit policy function was $b^{fb}(w) = 0.0339(w/y) + 51.87$. In Figure 6, we depicted the second-best benefit distributed to retiring workers as a function of $w$. When the market value of the assets of the company is below 495$y$, it is optimal to limit the benefit paid to the retiring workers to the guaranteed minimum benefit $b_{\text{min}} = 30y$. Otherwise, the benefit is increasing and concave in $w$. For very large values of $w$, the second-best benefit converges to the first-best solution. The second-best benefit is always smaller than its first-best level: $b^{sb}(w) \leq b^{fb}(w)$ for all $w$. The difference is particularly important when the fund’s reserves are too close to the legal minimum $w_{\text{min}} = 465y$.

This illustrates the fact that the financial reserves of the fund are used as a buffer stock à la Deaton (1991) to organize the intergenerational sharing of risk. The level of the buffer stock must be large enough above $w_{\text{min}}$ to make it feasible and efficient under the solvency constraint of the fund. This provides an additional incentives for the fund to reduce the distribution of benefits when $w - b_{\text{min}}$ is close to $w_{\text{min}}$. This reduction of the benefit paid by the fund in bad time is an effort made in the short run that is helpful to
restore the efficiency of the fund in the long run.

The second-best strategy to distribute dividends to the shareholders of the insurance company is depicted in Figure 7. When the minimum return constraint is not binding, the second-best dividend equals \((1 - k)/k = 5.26\%\) of the benefit paid to retiring workers, as explained in the previous section. When this constraint becomes binding, shareholders accept to drastically reduce their dividend.

4.4 The investment policy

The second-best investment in the risky asset in year \(t\) is a function of the market value of the assets of the fund at that time: \(\alpha_t = \alpha^{sb}(w_t)\). This optimal strategy is described in Figure 8. It should be compared to the first-best investment \(\alpha^{fb}(w) = a^*(w + K)\), which is characterized by a constant share of the risky asset in the total wealth of the fund, including the market value \(K\) of future contributions. Of course, this first-best strategy is not feasible in the second-best context, since the fund would still take risk when its financial reserves equals its critical level \(w_{min}\).

The second-best investment strategy is more precautionary than the first-best one in the sense that \(\alpha^{sb}(w)\) is smaller than \(\alpha^{fb}(w)\) for all \(w\). It is only when the fund’s reserves tends to infinity that the two investment strategies coincides. On the contrary, when the solvency constraint is likely to become binding soon, the fund drastically reduces its risk exposure. The solvency constraint reduces the ability of the fund to time-diversify the portfolio risk, thereby making it more risk-averse at low wealth levels. It is as if the fund would exhibit decreasing absolute risk aversion, as explained by Epstein (1983) and Gollier (2002).

In Figure 9, we have drawn the investment in the risky asset as a fraction of the financial reserves of the fund. It has an inverse U shape with a maximum at 60% around \(w = 1250y\). The maximum share invested in the risky asset is above the first-best level \(a^* = 39.6\%\) because the fund internalizes some of the future contributions in its true financial situation. As \(w\) increases, the effect of this implicit wealth of the fund vanishes, thereby explaining why \(\alpha^{sb}(w)/w\) decreases for large levels of \(w\). On the contrary, at low wealth levels, the effect of the solvency issue dominates and \(\alpha^{sb}(w)/w\) is increasing.
4.5 The dynamics of reserves accumulation

In the first-best solution, we have seen that the fund raises its total reserves \( w + K \) by a constant expected rate of 0.06% every year. Two contradictory effects affect this rate in the second-best context. First, because the distribution of benefits to retirees is less generous \((b^{sb}(w) < b^{fb}(w))\), the second-best fund accumulates reserves faster. This positive effect is particularly strong at low wealth levels. Second, the second-best investment strategy is more precautionary than the first-best one. This reduces the expected portfolio return of the fund. This negative effect on the rate of accumulation is particularly strong at low wealth levels. In Figure 10, we have drawn the expected rate of increase of the fund’s financial reserves as a function of \( w \). We see that the second-best expected growth rate is above the first-best one. This illustrates the fund’s willingness to escape the solvency problem that would inhibit the intergenerational risk-sharing.

Let us consider the effect of a negative excess return of \( x = -10\% \) of the risky asset on the flows generated by the fund. Suppose first that this negative shock occurs in year \( t = 0 \), when \( w = w_0 = 884y \). From this, the fund paid \( b_0 = 61.0y \) to new retirees and \( d_0 = 3.2y \) to shareholders. Because it collected 30y as contributions from workers, the fund’s reserves amounted to 849.8y, from which \( \alpha^{sb} = 504y \) has been invested in the risky asset. The return of the fund’s portfolio at the end of year \( t = 0 \) is thus equal to \( 0.02(849.8y) - 0.1(504y) = -33.4y \). The financial reserves at the beginning of the second year equals \( w_1 = 816.3y \), a 7.6% reduction from the previous year. It yields a reduction of the pension benefit to the generation retiring at the beginning of year \( t = 1 \) to \( b_1 = 57.8y \), a 5.2% reduction compared to the pension paid to the previous generation. The demand for the risky asset is reduced by 10.5%. It takes approximately 20 years for the fund to reconstitutes its reserves, assuming that the risky asset yields its mean return every year. In Table 2, we computed these values when the adverse shock occurs in other circumstances. The net effect of the financial shock on the fund reserves is increasing in the market value of its assets. This illustrates the fact that the fund takes proportionally more portfolio risk at high wealth levels.
<table>
<thead>
<tr>
<th></th>
<th>$w = 600y$</th>
<th>$w = 884y$</th>
<th>$w = 2000y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w/w$</td>
<td>4.8%</td>
<td>7.6%</td>
<td>7.7%</td>
</tr>
<tr>
<td>$\Delta b/b$</td>
<td>4.9%</td>
<td>5.2%</td>
<td>5.3%</td>
</tr>
<tr>
<td>$\Delta \alpha/\alpha$</td>
<td>20.2%</td>
<td>10.4%</td>
<td>6.6%</td>
</tr>
</tbody>
</table>

Table 2: The effect of a negative excess return $x = -10\%$ on reserves, pension benefits and the demand for stocks.

In Figures 11 to 14, we represented the five possible histories of the fund over 100 years, using 5 random scenarios of stock returns. In four of these scenarios, the fund is able to escape the solvency issue by accumulating enough reserves to share risk efficiently. In the worse scenario, a succession of financial losses pushes reserves close to their legal minimum level ($495y$). This forces the fund to reduce its portfolio risk, together with the pension benefits and dividends.

### 4.6 Welfare analysis

The discounted value of the flow of generational expected utilities at date $t = 0$ equals $U^{sb}$. As before, we measure the certainty equivalent constant benefit $B^{sb}$ distributed to all future generations that yields the same intergenerational welfare: $U^{sb} = u(B^{sb})/(1 - \beta)$. We obtain $B^{sb} = 55.82y$. This is in between the certainty equivalent pension $B^{aut} = 51.13y$ obtained in autaracy, and the certainty equivalent pension $B^{fb} = 67.89y$ obtained in the first-best context. The second-best pension system improves welfare, but the constraints imposed to it imply that only one-third of the potential gain from intergenerational risk-sharing can be extracted. In terms of certainty equivalent return on savings, we obtain $r^{sb} = 3.75\%$, which is in between $r^{aut} = 3.25\%$ and $r^{fb} = 4.85\%$.

An alternative approach consists in estimating the welfare of any specific generation conditional to the fund’s reserves when this generation starts contributing to the pension fund. Parallel to what we have done in the first-best context, we define the conditional certainty equivalent benefit $b^{sb}(w)$ as

$$u(b^{sb}(w)) = E \left[ u(b_{t+n}) \mid w_t = w \right].$$

The right-hand side of this equality can be numerically estimated by backward induction. The outcome of these computations is depicted in Figure
15. We see that $b^{sh}(w)$ is smaller than $b^{sh}(w)$ for all $w$. This means that the second-best fund is less attractive than the first-best fund for new workers at all reserves levels, because of the inefficient intergenerational risk-sharing. We see that new workers prefer to save on their own when the fund’s reserves at the time of their first contribution is less than $w^{sus} = 843y$. This is called the ”minimum sustainable reserve”. Notice that the initial reserve $Y_0 + e_0 = 884y$ exceeds the minimum sustainable reserve, so that the fund is attractive for the $n$ founding generations. However, there is no guarantee that the fund’s assets value will remain permanently above $w^{sus}$. If the participation to the fund is on a voluntary basis, when $w_t$ is less than $w^{sus}$, the new generation entering in year $t$ would not accept to contribute to the fund unless a tax incentive for long-term savings is offered. For example, in the most critical situation with $w_t = 500y$, the certainty equivalent benefit distributed by the second-best fund is equal to $b^{sh}(500y) = 41.4y$, which is 19% smaller than the certainty equivalent benefit obtained in autarchy. A tax incentive of this order would be enough to make the second-best pension scheme sustainable.

5 Sensitivity analysis

In this section, we perform some sensitivity analyses of the second-best pension scheme. The parameters of the model are $\beta$ (the discount factor), $R_{min}$ (the guaranteed minimum gross return), $R$ (the risk-free rate), $\bar{x}$ (the distribution of the stock returns), $\gamma$ (relative risk aversion), and $e_0$ (the initial equity). As explained in section 4.2, there is a one-to-one relationship between $e_0$ and $v_0$ (equation (14), and between $v_0$ and the share $k$ of the yearly payment of the fund that is allocated to the retiring generation, the remaining $1 - k$ being paid to shareholders. It is much easier numerically to fix $k$ as an exogenous choice variable, and to derive $e_0$ that is compatible to this $k$ at equilibrium. We have seen earlier that our benchmark initial equity $e_0 = 100y$ is compatible with the distribution of $k = 95\%$ of the yearly fund’s payment to workers. In our sensitivity analysis, we fix $k = 95\%$ and we select the initial equity $e_0$ that is compatible with this $k$. Table 3 summarize our findings. The different lines of this table are self-explanatory, except the line referred to as $E(w_{t+1} - w_t)/w_t$. As seen in Figure 10, the expected yearly

\[^{a}\text{Notice that } w^{sus} \text{ is defined by } u(b^{sh}(w^{sus})) = u^{aut}.\]
change in the fund’s assets value depends upon the state variable $w$. On this line, we report its asymptotic value $R(1 - m)[1 + a^*E\ddot{x}] - 1.$

<table>
<thead>
<tr>
<th></th>
<th>Second best benchmark</th>
<th>$\beta = 0.94$</th>
<th>$\beta = 0.93$</th>
<th>$R_{\text{min}} = 100%$</th>
<th>$R_{\text{min}} = 95%$</th>
<th>$k = 95%$</th>
<th>$k = 85%$</th>
<th>$\ddot{x}$</th>
<th>$\ddot{x} - 2%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^{ab}$</td>
<td>3.76%</td>
<td>3.82%</td>
<td>4.0%</td>
<td>3.94%</td>
<td>2.66%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_0 + e_0$</td>
<td>884$y$</td>
<td>882$y$</td>
<td>889$y$</td>
<td>1131$y$</td>
<td>725$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>507$y$</td>
<td>489$y$</td>
<td>613$y$</td>
<td>674$y$</td>
<td>224$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>61.2$y$</td>
<td>63.0$y$</td>
<td>65.2$y$</td>
<td>64.2$y$</td>
<td>51.8$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_0$</td>
<td>3.2$y$</td>
<td>3.3$y$</td>
<td>3.4$y$</td>
<td>11.3$y$</td>
<td>2.7$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(w_{t+1} - w_t)/w_t$</td>
<td>0.06%</td>
<td>-0.16%</td>
<td>0.06%</td>
<td>0.06%</td>
<td>-0.62%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w^{sus}$</td>
<td>843$y$</td>
<td>883$y$</td>
<td>786$y$</td>
<td>1047$y$</td>
<td>776$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Sensitivity analysis of the second-best solution.

- Reduction of $\beta$ from 0.94 to 0.93: The current generations are favored by this reduction of the discount factor. The benefit paid to the retiring workers during the first year of the fund is increased, together with the dividend paid to shareholders. Because risk are less efficiently allocated across generations, the fund reduces its exposure to portfolio risk. All these changes converge towards the fact that the expected yearly change in the fund’s assets value be reduced with respect to the benchmark. In fact, the expected change becomes negative. Financial reserves converge over time to $w_{\text{min}}$. Because of its generous benefits paid early to retiring workers, the fund is beneficial for intergenerational risk-sharing only in the short and medium terms. In the long term, the financial reserves of the fund cannot be used as a buffer stock to smooth shocks across generations, because of the solvency constraint.

- Reduction of $R_{\text{min}}$ from 100% to 95%: Reducing the guaranteed minimum return on savings is good for the intergenerational sharing of risks. This change has a sizeable impact on the certainty equivalent return of the pension scheme. Because shareholders are less exposed to the downside financial risk, the equilibrium initial equity $e_0$ is increased. The optimal portfolio risk exposure is also increased, because this risk is shared more efficiently with future generations. This yields an increase in the benefits paid to new retirees and to shareholders.
• Reduction of $k$ from 95% to 85%: Increasing the share of the fund’s portfolio return accruing to shareholders raises the equilibrium equity initially invested in the fund by shareholders. Because the initial value of the fund’s assets is much above the legal minimum to satisfy the solvency constraint, the fund is in a much better situation to organize an efficient intergenerational risk-sharing. The certainty equivalent return on savings is thereby improved compared to the benchmark. The share of the fund’s assets invested in stocks, the benefit paid to new retirees and the dividend paid to shareholders are all increased.

• Deterministic reduction of stocks returns by 2%: This change in expectations dramatically reduces both the investment in the risky asset and the certainty equivalent return on savings. Because of the reduced return of savings, the willingness to accumulate wealth over time is reduced. In fact, a decumulation process arises under these values of the parameters, as in the case of the reduction of $\beta$.

6 Concluding Remarks

The idea that our market economies cannot organize an intergenerational sharing of risk without a public intervention is widely accepted. However, up to our knowledge, there has been no attempt to measure the welfare loss of this market incompleteness. In addition, there has been no attempt to propose operational rules for a pension fund to optimize both the sharing of risk across generations and the dynamic portfolio management of this fund. This paper tried to fill in these two gaps in the literature. Compared to a purely individual retirement system, organizing a first-best intergenerational risk sharing as an effect on welfare that is equivalent to an increase in more than 1.5 percentage points of the return on retirement savings. However, this welfare gain is limited to 0.5 to 1.0 percentage points when solvency constraints are imposed to the pension fund to reinforce the long-term political sustainability of the scheme.

Many advocates of the preservation of the PAYG system argued that this system, contrary to a funded system, is able to implement efficient intergenerational risk sharing. The current trends towards a more funded system in many countries raises some doubt about this statement. In this paper, we
have shown that a cleverly built, transparent funded system can do much in favor of risk-sharing efficiency under a limited public intervention taking the form of some tax incentives. Our work is also useful to refine an argument long used by proponents of radical changes in social security schemes. Under their view, the PAYG system should be abolished due to its low rate of return. The problem is to determine to which rate of return should it be compared. If it is compared to the real risk-free rate, which has been around 1.5% in the U.S. and around -1% in France during the XXth century, the argument is not very strong. If it is compared to the average real return on equity, which has been around respectively around 7% and 4% in the U.S. and in France, the argument would be much stronger. Some proponents of the funded system claim that this alternative comparison is the relevant one, because of the ability of the pension fund to time-diversify risk. Using a calibration based on U.S. financial data, we have shown that this argument is only partially true, and that the implementation of the realistic second-best pension scheme has a certainty equivalent rate of return around 3.75% and 4%.

This paper could obviously be improved in many directions. First, we assumed that the risk-free rate is constant over time and that there is no mean-reversion in stocks returns. We recognize that these two assumptions are important, but quite unrealistic. Life insurers are very sensitive to the interest rate risk and this should be treated formally in a future extension of this work. Introducing predictable changes in the two assets returns will make the second-best benefit policy function dependent of the additional state variables of the system, in particular of the current interest rate. Second, we considered for simplicity a defined contribution scheme. It would be useful to see how a more flexible contribution rule would be useful for a better sharing of risk over time. Third, one could consider more sophisticated portfolio insurance rules, such as the one consisting in “locking in” successive benefits to active members of the fund. This would be a third-best approach.

References

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7For a synthesis of financial returns during the last century, see Dimson, Marsh and Staunton (2000).
in the spirit of Arrow, Debreu, and Rawls, with applications to social security design, Harvard Institute of Economic Research DP 1921.


Figure 2: Real excess return of the SP500.

Figure 3: Evolution of the financial reserves under the optimal dynamic portfolio strategy in autarcy, for 10 random scenarios of asset returns.
Figure 4: Evolution of the share of financial reserves invested in stocks under the optimal dynamic portfolio strategy in autarcy, for 10 random scenarios of asset returns.
Figure 5: Ten random scenarios on the evolution of financial reserves under the first-best distribution and investment strategy. The horizontal line is the minimum reserves under which the young generation prefers the autarcic solution.
Figure 6: The second-best benefit function compared to the first-best one (dashed line).

Figure 7: The dividend function in the second-best model.
Figure 8: The second-best investment in the risky asset as a fraction of the total wealth of the fund $w + K$. The dashed line describes the first-best investment strategy.

Figure 9: The second-best share of the fund’s wealth invested in the risky asset. The dashed curve describes the first-best strategy.
Figure 10: The expected yearly rate of increase of reserves in the second-best context (plain curve) and in the first-best one (dashed curve).

Figure 11: Evolution of the market value of the financial assets of the fund.
Figure 12: Distribution of pension benefits.

Figure 13: The share of reserves invested in the risky asset.
Figure 14: Distribution of dividends.

Figure 15: The certainty equivalent benefits conditional to the fund’s reserves at the beginning of the contribution period.